

# The third Zemach moment and the size of the proton

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To resolve the puzzle of the proton size raised from the recent result of muonic hydrogen Lamb shift, De Rújula has proposed that a large value of the third Zemach moment  $\langle r_p^3 \rangle_{(2)}$  of the proton to be the solution. His suggestion has been criticized by many groups based on the  $ep$  scattering data at low  $Q^2$  regime. However, if there is a “thorn” or “lump” in the electric form factor of the proton  $G_E(Q^2)$  at extremely low  $Q^2$  regime, then the third Zemach moment  $\langle r_p^3 \rangle_{(2)}$  would be as large as De Rújula suggested. In this article, we show that the existence of such a “thorn” or “lump” has not been completely excluded, although tightly restricted, by the current data of  $ep$  elastic scattering. We also suggest a more sophisticated global fitting procedure of  $G_E(Q^2)$  for the future fitting.

## I. INTRODUCTION

The issue of the charge radius of the proton has attracted a lot of attention, since the charge radius extracted from the Lamb shift of muonic hydrogen has been reported to be  $0.84184(67) fm$  [1]. This result is significantly smaller than the previous value of CODATA [2],  $\sqrt{\langle r_p^2 \rangle}(\text{CODATA})=0.8768(69) fm$  and the one extracted from  $ep$  elastic scattering data  $\sqrt{\langle r_p^2 \rangle}(\text{ep})=0.879(5)_{stat}(4)_{syst}(2)_{model}(4)_{group} fm$  [3]. De Rújula [4, 5] has pointed out that the small value of the charge radius reported in [1] based on the assumption that the electric form factor of the proton  $G_E(Q^2)$  is the dipole form. Hence the original QED formula,

$$L^{th}(meV) = 209.9779 - 5.2262\langle r_p^2 \rangle + 0.00913\langle r_p^3 \rangle_{(2)}, \quad (1)$$

is reduced into

$$L^{th}(meV) = 209.9779 - 5.2262\langle r_p^2 \rangle + 0.0347\langle r_p^2 \rangle^{3/2}, \quad (2)$$

because

$$[\langle r_p^3 \rangle_{(2)}]^2 = \frac{3675}{256}[\langle r_p^2 \rangle]^3, \quad (3)$$

when  $G_E(Q^2)$  is the dipole form. (Note that in the above equations, the units of  $\langle r_p^2 \rangle$  and  $\langle r_p^3 \rangle_{(2)}$  are  $\text{fm}^2$  and  $\text{fm}^3$ , respectively.) Since the experimental result is  $L^{exp}=206.2949 \pm 0.0032$  meV, accordingly they concluded that the value of the charge radius is 0.84184 fm [1]. However, De Rújula has argued that there is no reason to believe  $G_E(Q^2)$  to be the dipole form. Instead he suggested that the proton may own a large third Zemach moment about  $36.59 \text{ fm}^3$ , which is about fifteen times larger than the value from Eq.(3). If so the value of the charge radius extracted from Eq.(1) will agree with the CODATA value well. Furthermore he has employed some toy model of the  $G_E(Q^2)$  to obtain  $\langle r_p^3 \rangle_{(2)}=36.59 \text{ fm}^3$ . By this way one is able to resolve the proton size puzzle [4, 5].

The other attempts to resolve this puzzle, for example, to recalculate the polarizability contribution [6] or to estimate the non-perturbative effect [7], and to test the possibility of the existence of the new particle between the proton and the muon [8], have been not very successful so far. The new corrections they have found are usually too small (The only exception is so-called off-mass-shell effect advocated by [9]). Therefore the simple solution suggested by De Rújula seems to be worthy of further investigation.

However, the proposal of De Rújula has been severely criticized by several groups [10–12]. First, the toy model used by De Rújula has been indeed ruled out by the recent experimental data. Furthermore, those groups have argued that such a large value of the third Zemach moment cannot accommodate the current data of  $ep$  elastic scattering. What they did, instead, is to adopt several widely used parametrizations of  $G_E(Q^2)$  to calculate the correspondent third Zemach moment  $\langle r_p^3 \rangle_{(2)}$  and presented them to be far smaller values than the one obtained by De Rújula. At first glance, this objection looks very convincing. However, as De Rújula already pointed out, the value of  $\langle r_p^3 \rangle_{(2)}$  is extremely sensitive to the behaviour of  $G_E(Q^2)$  in very low  $Q^2$  regime. Because there is no data between  $Q^2 = 0$  to  $Q^2 = Q_{min}^2$ . Therefore he argued the slim possibility of large third Zemach moment may not completely excluded yet [5].

But one can provide a counterargument as follows: the extrapolation of  $G_E(Q^2)$  between  $Q^2=0$  and  $Q^2=Q_{min}^2$  should be very reliable. Because the value of  $G_E$  at  $Q^2 = 0$  must be one due to the fact that the electric charge of the proton is  $+e$ , and its derivative  $\frac{dG_E(Q^2)}{dQ^2}$  at  $Q^2=0$  is also severely constrained by the CODATA value of  $\langle r_p^2 \rangle$ . Thus the extrapolation of  $G_E(Q^2)$  from  $Q^2=Q_{min}^2$  to  $Q^2=0$  is supposed to be adequate enough to determine the value of  $\langle r_p^3 \rangle_{(2)}$ . However this counterargument has one loophole. If there appears a "thorn" or

”dip” in  $G_E(Q^2)$  between  $Q^2=0$  and  $Q^2=Q_{min}^2$  then the parametrizations previously used [13, 14] will no longer be able to produce an accurate value of  $\langle r_p^3 \rangle_{(2)}$ . Naturally one should ask that whether there exists a  $G_E(Q^2)$  with “thorn” or “lump” which can generate a large  $\langle r_p^3 \rangle_{(2)}$ , and at the same time, accommodate the existing  $ep$  scattering data. In particular, recently a measurement of the cross section of the elastic  $ep$  scattering has been carried out at Mainz, ranged from  $Q^2 = 0.004 \text{ GeV}^2$  to  $1 \text{ GeV}^2$  with the statistical errors below 0.2% [3]. Their data has shown no sign of any “bump”. It makes any attempt to obtain large  $\langle r_p^3 \rangle_{(2)}$  by adding “thorn” at  $G_E(Q^2)$  to be very difficult. But in this article we will explicitly show that such a task is indeed difficult but not totally impossible.

The outline of this article is as follows. We first review the relationship between the third Zemach moment and the electric form factor  $G_E(Q^2)$ . Next we combine the “thorn” and some parametrizations of  $G_E(Q^2)$  to calculate the third Zemach moment and learn the relation between the height, width and peak position of the “thorn” and the third Zemach moment of the proton. Then we explicitly show that one can combine our ansatz of “thorn” with the inverse-polynomial fit used in [3] to obtain a large third Zemach moment as De Rújula has suggested. At the same time the combined ansatz deviates from the original inverse-polynomial fit less than 0.2%. Finally we present our conclusions and outlooks.

## II. ZEMACH MOMENT AND THE ELECTRIC FORM FACTORS

The conventional proton charge density is defined as the Fourier transform of the electric form factor  $G_E(Q^2)$  in the Breit frame,

$$\rho_p(r) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} G_E(\mathbf{q}). \quad (4)$$

Here  $Q^2 = -q^2 = |\vec{q}|^2 - q_0^2$  which is equal to  $|\vec{q}|^2$  in the Breit frame. We use the notation  $\mathbf{q} = |\vec{q}|$ . The following quantities are defined as

$$\langle r_p^n \rangle = \int d^3r r^n \rho_p(r). \quad (5)$$

From this definition one can easily deduce that  $G_E(0) = 1$  because the 0-th moment  $\langle r_p^0 \rangle = 1$  and  $\frac{dG_E(\mathbf{q}^2)}{d\mathbf{q}^2}|_{\mathbf{q}^2=0} = -\frac{1}{6}\langle r_p^2 \rangle$ . On the other hand the  $n$ -th Zemach moment is defined as

$$\langle r^n \rangle_{(2)} = \int d^3r r^n \rho_2(r), \quad (6)$$

where  $\rho_2(r)$  is defined as

$$\rho_2(r) = \int d^3r' \rho_p(\mathbf{r}') \rho_p(\mathbf{r}' - \mathbf{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} G_E^2(\mathbf{q}). \quad (7)$$

After some algebra one obtains the following result [15],

$$\langle r_p^3 \rangle_{(2)} = \frac{48}{\pi} \int_0^\infty \frac{d\mathbf{q}}{\mathbf{q}^4} \left[ G_E^2(\mathbf{q}) - \frac{\mathbf{q}^2}{3} \langle r_p^2 \rangle - 1 \right]. \quad (8)$$

It is obvious that the third Zemach moment of the proton is dominated by the  $G_E(Q^2)$  at very low  $Q^2$ . The crucial issue here is whether there exists one form of  $G_E(Q^2)$  which is able to produce large  $\langle r_p^3 \rangle_{(2)}$  and at the same time accommodate the current data of  $ep$  elastic scattering.

### III. RELATION BETWEEN THE THORN IN $G_E$ AND THE THIRD ZEMACH MOMENT

In this section we assume that there is some “thorn” or ”lump” appearing in the  $G_E(Q^2)$  in the very low  $Q^2$  regime. We expect such a pathological structure to produce a large third Zemach moment. Here we express the electric form factor as follows,

$$G_E(Q^2) = G_E^{(R)}(Q^2) + \Delta G_E(Q^2), \quad (9)$$

where  $G_E^{(R)}(Q^2)$  is some parametrization from the global fitting of the  $ep$  scattering data. On the other hand  $\Delta G_E(Q^2)$  denotes the ”thorn” on the electric form factor. Naively, one may think that it is easier to simply add a triangle function with the height  $H$  and the width  $W$ , whose peak is located at  $Q_{peak}^2$ . However such a choice will cause a serious problem. One can calculate the associated charge density  $\Delta\rho(r)$  by making the Fourier transform of  $\Delta G_E(Q^2)$ , then calculating its contribution to  $\langle r_p^2 \rangle$ . However, if the triangle function is chosen then its corresponding  $\Delta\langle r_p^2 \rangle$  calculated by

$$\Delta\langle r_p^2 \rangle = \int d^3r \Delta\rho(r) r^2, \quad (10)$$

is actually divergent! It is due to the fact of the corresponding  $\Delta\rho(r)$  actually converges slower than  $1/r^4$ . Hence one has to make judicious choice of the ”thorn” function so that  $\langle r_p^2 \rangle$  can be kept finite. On the other hand, here we still want to employ the widely used parametrizations of  $G_E(Q^2)$  whose value at the  $Q^2 = 0$  have been fixed. As a result

$\Delta G_E(Q^2 = 0)$  and  $\frac{d\Delta G_E}{dQ^2}(Q^2 = 0)$  both have to be negligible. Moreover the influence of  $\Delta G_E(Q^2)$  has to be able to be ignored when  $Q^2 \geq Q_{min}^2$ . One needs figure out some function form satisfying the above criteria. Here we present our choice as follows,

$$\Delta G_E(Q^2) = K_1 \exp \left[ -\frac{(Q^2 - K_2)^2}{K_3^4} \right]. \quad (11)$$

Here  $K_1$  is dimensionless and the unit for  $K_2$  and  $K_3$  is GeV.  $K_1$ ,  $K_2$  and  $K_3$  denote the height, the position of the peak and the width, respectively.

To explain the result of muonic hydrogen Lamb shift one needs show that the following quantity

$$\Delta L(\text{meV}) = L^{\text{theory}} - L^{\text{exp}} = 209.9779 - 5.2262\langle r_p^2 \rangle + 0.00913\langle r_p^3 \rangle_{(2)} - 206.2949, \quad (12)$$

to be smaller than the experimental uncertainty  $3 \times 10^{-3}$  meV. If we choose the parametrizations of [13] or [14] as our  $G_E^{(R)}$ , it is easy to pick up several parameter sets of  $K_{1,2,3}$  to satisfy all criteria. We list our parameter sets and their corresponding values of  $\langle r^2 \rangle$  and  $\langle r^3 \rangle_{(2)}$  in Table (I). One may wonder the values of  $\langle r_p^2 \rangle$  is somehow too small compared with the CODATA value:  $\langle r_p^2 \rangle(\text{CODATA})=0.753 \text{ fm}^2$ . The reason for it is because the parametrizations we used are the results of global fitting and their value of  $\langle r_p^2 \rangle$  are somehow small. For example,  $\langle r_p^2 \rangle(\text{Alberico})=0.750 \text{ fm}^2$  and  $\langle r_p^2 \rangle(\text{Kelly})=0.744 \text{ fm}^2$ .

	$G_E^{(R)}$	$K_1$	$K_2$ (GeV)	$K_3$ (GeV)	$\Delta L(\text{meV})$	$\langle r_p^2 \rangle$ (fm <sup>2</sup> )	$\langle r_p^3 \rangle_{(2)}$ (fm <sup>3</sup> )
I	Alberico	0.119185	0.08	0.0447214	$1.7 \times 10^{-4}$	0.745137	23.138
II	Alberico	0.0139929	0.08	0.053183	$1.5 \times 10^{-4}$	0.717153	7.11973
III	Alberico	0.283648	0.10	0.053183	$-2.85 \times 10^{-4}$	0.747693	24.5962
IV	Alberico	0.139056	0.10	0.0588566	$8.56 \times 10^{-6}$	0.735336	17.5272
V	Alberico	0.720091	0.12	0.053183	$1.79 \times 10^{-4}$	0.748308	24.9536
VI	Kelly	0.130982	0.08	0.0422949	$1.06 \times 10^{-6}$	0.742016	21.3496
VII	Kelly	0.101611	0.08	0.0447214	$-9.31 \times 10^{-6}$	0.739647	19.9926
VIII	Kelly	0.243554	0.10	0.053183	$1.06 \times 10^{-4}$	0.741824	21.2389
IX	Kelly	0.118405	0.10	0.0588566	$-8.5 \times 10^{-4}$	0.731308	15.2113
X	Kelly	0.627337	0.12	0.053183	$4.6 \times 10^{-6}$	0.742354	21.5437

TABLE I: Our chosen parameter sets and the values of their corresponding  $\langle r_p^2 \rangle$  and  $\langle r_p^3 \rangle_{(2)}$ .

Unfortunately the above results have all been excluded by the recent Mainz low  $Q^2$  data[3]. Nevertheless, we have observed several important facts from the Fig (1). First, if we make the position of peak,  $K_2$ , more close to the  $Q^2=0$ , the height of the peak will be smaller with the same width. However, if  $K_2$  becomes too small, it will produce relatively large  $\Delta\langle r^2 \rangle$ , which is negative, thus the resultant  $\langle r^2 \rangle$  is much smaller than the CODATA value. The second important fact is as follows. With the same  $K_2$ , the height  $K_1$  decreases as the width  $K_3$  increases. i.e., when the peak is less sharp and the height becomes smaller. Thirdly, we also find that the result is not very sensitive to the choice of the  $G_E^{(R)}(Q^2)$  parametrizations as shown by the Table (I). These facts will instruct us to construct more realistic ansatz of  $G_E(Q^2)$  as shown in the next section.

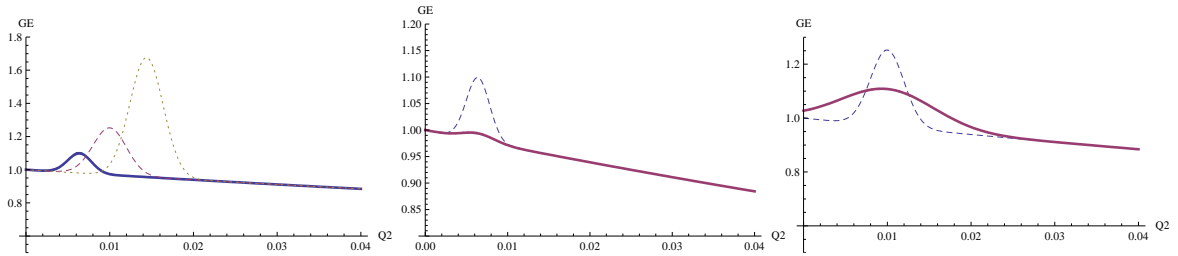


FIG. 1: The unit in  $x$  axes is  $\text{GeV}^2$ . (Left) The  $G_E(Q^2)$  in (II)(solid line), (III)(dashed line) and (V)(dotted line). Three curves correspond to the same  $K_3=0.053 \text{ GeV}$  but their values of  $K_2$  are  $0.08 \text{ GeV}$ ,  $0.10 \text{ GeV}$  and  $0.12 \text{ GeV}$  respectively. (Middle) The  $G_E(Q^2)$  in (I) (dashed line), (II)(solid line) with the common value of  $K_2=0.08 \text{ GeV}$ . But their corresponding values of  $K_3$  are  $0.0447 \text{ GeV}$  and  $0.0532 \text{ GeV}$ , respectively. (right) The  $G_E(Q^2)$  in (III)(dashed line), (IV) (solid line) with the common value of  $K_2=0.10 \text{ GeV}$ . But their corresponding values of  $K_3$  are  $0.0532 \text{ GeV}$  and  $0.0589 \text{ GeV}$ , respectively.

#### IV. INVERSE-POLYNOMIAL FIT WITH A “THORN” OF $G_E$

We have learned how to increase the third Zemach moment by adding the “thorn” in the previous section. Here we need construct a parametrization to accommodate the recent Mainz low  $Q^2$  data with  $Q_{min}^2=0.004 \text{ GeV}^2$  with the uncertainty below  $0.2\%$ . In this section we show that one is able to combine our ansatz of “thorn” with the inverse-polynomial fit used in [3, 16] to make the difference between  $L^{th}$  and  $L^{exp}$  to be smaller than  $3 \times 10^{-3} \text{ meV}$ .

At the same time the combined ansatz deviates from the original inverse-polynomial fit less than 0.2%. The inverse-polynomial fit has been used in [3, 16]. Its explicit form is given as

$$G_E^{inv-poly}(Q^2) = \frac{1}{1 + \sum_{i=1}^7 a_i Q^{2i}}. \quad (13)$$

with

$$\begin{aligned} a_1 &= 3.3615, a_2 = -3.0343, a_3 = 29.6677, a_4 = -85.6169, \\ a_5 &= 130.7053, a_6 = -101.5145, a_7 = 34.2926. \end{aligned} \quad (14)$$

Note that  $G_E^{inv-poly}(Q^2)$  generates  $\langle r^3 \rangle_{(2)} = 2.96667 \text{ fm}^3$ . To combine this fit with the ansatz in Eq.(11), one needs to guarantee that  $G_E(0) = 1$ . Hence we modify  $G_E^{inv-poly}$  into the following one,

$$G_E^{mod}(Q^2) = (1 - \Delta G_E(0))G_E^{inv-poly}(Q^2) + \Delta G_E(Q^2). \quad (15)$$

Employing the ansatz in Eq.(15), one can make  $\Delta L$  to be smaller than  $3 \times 10^{-3} \text{ meV}$  with the chosen parameters listed in the Table (II). The value of  $\langle r_p^2 \rangle$  is a little larger than CODATA value but still be reasonable. To accommodate the very precise Mainz low  $Q^2$  data, it is necessary to make  $K_2$  very small. The width  $K_3$  is about only half of the value used in the previous section. The height  $K_1$  is only few percents of the ones in Table (I). However such a “lump” generates a very large  $\langle r_p^3 \rangle_{(2)}$ . It shows that the third Zemach moment is extremely sensitive to the detail of the electric form factor  $G_E(Q^2)$  at very low  $Q^2$  regime.

$K_1$	$K_2 \text{ (GeV)}$	$K_3 \text{ (GeV)}$	$\Delta L \text{ (meV)}$	$\langle r_p^2 \rangle \text{ (fm}^2\text{)}$	$\langle r_p^3 \rangle_{(2)} \text{ (fm}^3\text{)}$
-0.0016962	0.001	0.0221336	$5.95 \times 10^{-4}$	0.787349	47.3646

TABLE II: Our chosen parameters and the values of  $\langle r_p^2 \rangle$  and  $\langle r_p^3 \rangle_{(2)}$ .

Moreover we define the following quantity to characterize the difference of our modified fit and the original inverse-polynomial fit.

$$R(Q^2) = \left[ \frac{G_E^{mod}(Q^2) - G_E^{inv-poly}(Q^2)}{G_E^{inv-poly}(Q^2)} \right]. \quad (16)$$

The left panel of Fig.(2) shows that the value of  $R(Q^2)$  is ranged from 0 to 0.17%. It is due to the fact  $\Delta G_E(Q^2 = 0) = -1.6 \times 10^{-3}$ . When  $Q^2$  is large enough,  $\Delta G_E \sim 0$  and  $G_E^{mod}(Q^2) \sim (1 - \Delta G_E(0))G_E^{inv-poly}(Q^2)$ . One can also observe the curve of  $G_E^{mod}(Q^2)$ . It is

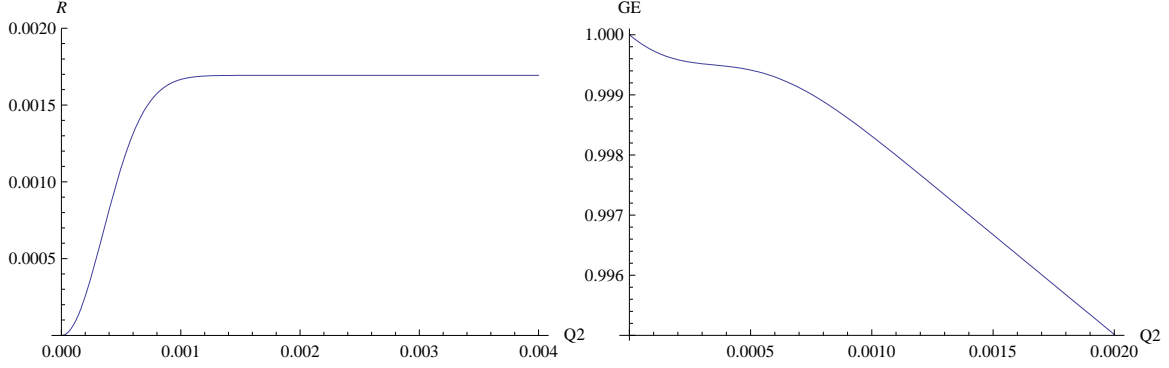


FIG. 2: (left) The value of  $R(Q^2)$  defined in the text. (right) The curve of  $G_E^{mod}(Q^2)$ .

a very smooth “lump” hidden at the extreme low  $Q^2$  regime. The shape of this “lump” is depicted in the right panel of Fig.(2). It is very smooth as one can observe from the plot.

One may frown on our result here and argue that our result cannot accommodate the Mainz data satisfactorily. Indeed the ansatz in Eq.(15) is quite a simple way to add a “lump” to the existing fit. However, we emphasize that even with such a simple ansatz one can still embed a “lump” at  $G_E(Q^2)$  and produce a large  $\langle r_p^3 \rangle_{(2)}$ . We believe it to be promising to improve the result with better agreement with the data by employing more sophisticated ansatz instead of the one we used here. We leave it for our future publication [17].

## V. CONCLUSION AND OUTLOOK

In this article, we show that the third Zemach moment becomes large if there is a “thorn” or “lump” at very low  $Q^2$  regime. Furthermore, our study show that it is possible to construct a parametrization of  $G_E(Q^2)$  which can accommodate the existent  $ep$  elastic scattering data and, at the same time, generate a large  $\langle r_p^3 \rangle_{(2)}$  to explain the Lamb shift of the muonic hydrogen.

In this work we limit ourselves to combine the existent parametrizations of  $G_E^{inv-poly}(Q^2)$  and a simple ansatz denoting the “thorn” by a very simple way in Eq.(15). In principle one should use the following ansatz,

$$G_E(Q^2) = \frac{a_1 + a_2 Q^2}{1 + a_3 Q^2 + a_4 Q^4 + a_6 Q^6} + b_1 \exp \left[ \frac{-(Q^2 - b_2^2)^2}{b_3^4} \right], \quad (17)$$

to fit the  $ep$  scattering data globally. There are two relations between those parameters,

$$1 = a_1 + b_1 \exp \left[ \frac{-b_2^4}{b_3^4} \right], \quad (18)$$



$$\frac{\langle r_p^2 \rangle (CODATA)}{6} = -a_2 + a_3 + \frac{2b_1 b_2^2}{b_3^4}. \quad (19)$$

Here we have four new parameters  $a_1$  and  $b_{1-3}$  with several constrains such as Eq. (18) and Eq.(19). The third constraint is the resultant result of Eq.(8) has to be around  $36 \text{ fm}^3$ . Using the above parametrization, it is likely that one can pick up a suitable parameter set to accommodate the existent  $ep$  elastic scattering data. The result by default can explain the Lamb shift of both electronic and muonic hydrogen. We leave this task for our future publication [17].

Although phenomenologically a large third Zemach moment is possible, nevertheless, there are still many challenges from theory side. For example, the very low  $Q^2$  behaviour of  $G_E(Q^2)$  is supposed to be dominated by the chiral physics. Namely the pion cloud plays the crucial roles in the low energy regime and one can apply Chiral Perturbation Theory ( $\chi$  PT) to calculate the electric form factor there[18]. The  $\chi$  PT result of  $\langle r_p^3 \rangle_{(2)}$  is about  $2 - 3 \text{ fm}^3$ , which is much smaller than ours. We also notice the most recent estimate made by [19], their conclusion disagrees with us. The reason is they insist to adopt the smooth  $\rho(r)$  which guarantees  $\langle r^n \rangle$  to be always finite. We only require the convergence of Eq.(8) and Eq.(10) only. These issues all remain open and need further studies.

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